Verification and search algorithms for causal DAGs

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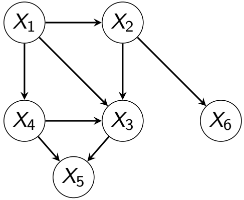
* Equal contribution

Motivation

Underlying data
generation process
(modelled as a DAG)

Observational data D

G



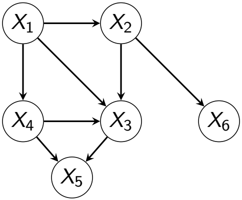
	X_1	X_2	X_3	X_4	X_5	X_6
Sample 1	0.3	0.4	0.1	-0.5	0.2	-0.3
Sample 2	0.1	1.2	0.6	-0.2	-0.1	-0.4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

e.g. $X_4 = f_4(X_1, \varepsilon_4)$

specific to node X_4

Motivation

G

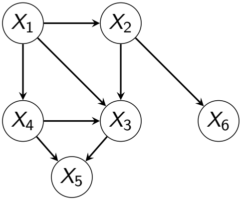


D

	X_1	X_2	X_3	X_4	X_5	X_6
Sample 1	0.3	0.4	0.1	-0.5	0.2	-0.3
Sample 2	0.1	1.2	0.6	-0.2	-0.1	-0.4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Motivation

G

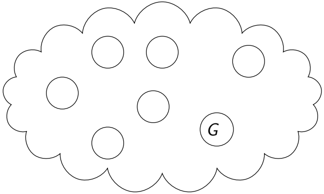


D

	X_1	X_2	X_3	X_4	X_5	X_6
Sample 1	0.3	0.4	0.1	-0.5	0.2	-0.3
Sample 2	0.1	1.2	0.6	-0.2	-0.1	-0.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Identify possible graphs
via conditional dependencies
(e.g. PC [SGSH00], GES [Chi02])

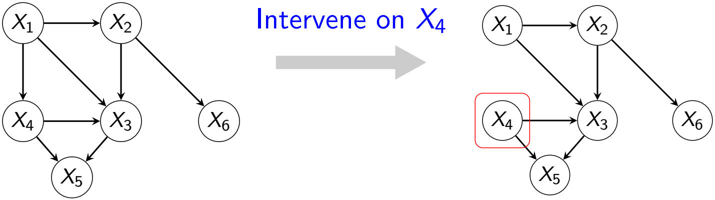
Equivalence
class of DAGs



Which is G ?

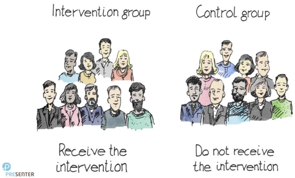
Two ways forward

1. Make model assumptions about the functional dependencies
   e.g. $X_4 = f_4(X_1, \varepsilon_4) = \alpha X_1 + \varepsilon_4$, where ε_4 is non-Gaussian
2. Perform interventions (Our focus)
   e.g. set $X_4 = 0.5$, then draw samples from the resulting intervened causal graph

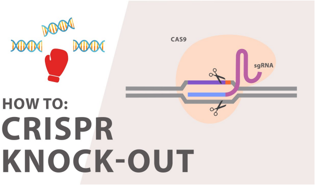


Interventions in real-life

Randomized controlled trials



Gene knockout experiments



Can be expensive to perform) Minimize number of interventions!

Image credit: https://i.ytimg.com/vi/vyVyg4Bf2JY/maxresdefault.jpg
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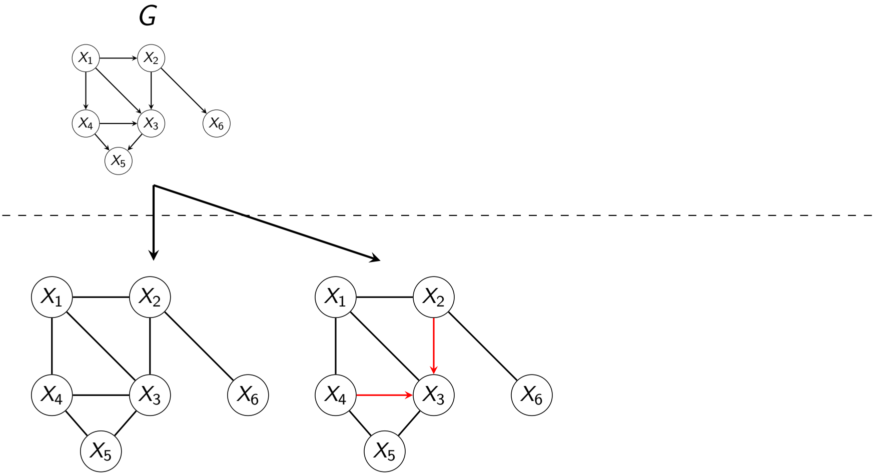
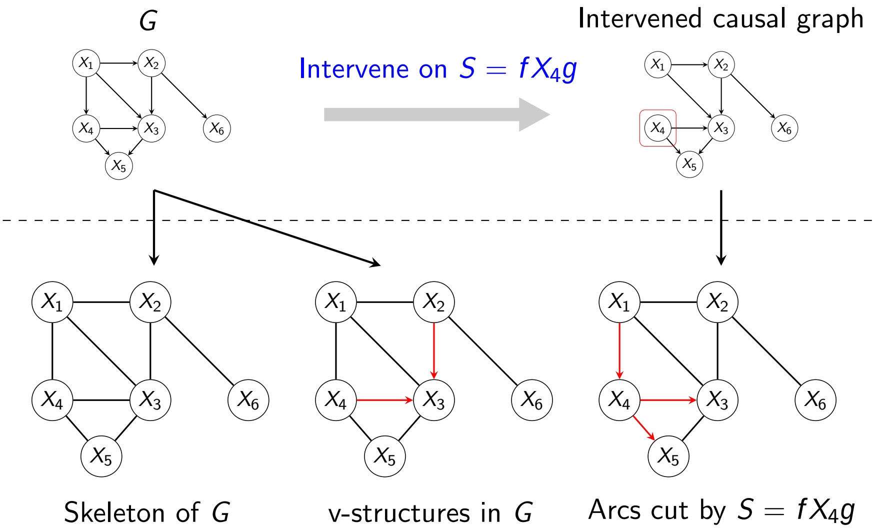
What can we learn about G from D and interventions?



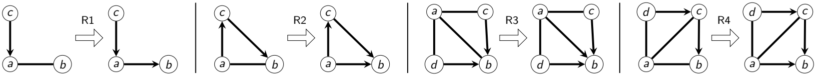
Skeleton of G v-structures in G

What can we learn about G from D and interventions?



Meek rules

Meek rules [Mee95]:
A set of 4 arc orientation rules that are *sound* and *complete*
(with respect to arc orientations with acyclic completion)



If b a,
then v-structure

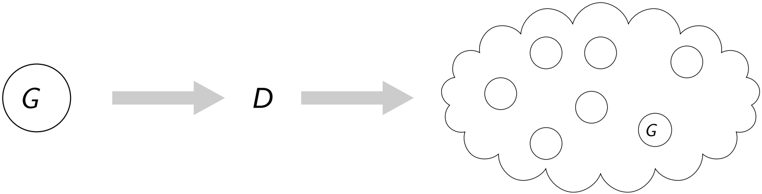
If b a,
then cycle

If b a, then the unoriented arcs would have been *oriented in the same way in all DAGs within the equivalence class* (via R2)

Meek rules converge in polynomial time [WBL21, Algorithm 2].

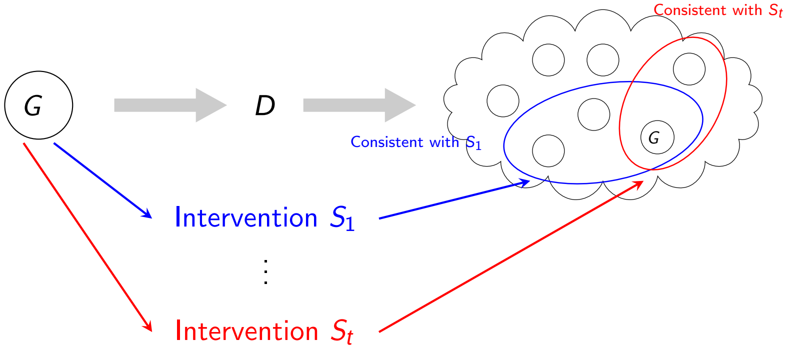
Problem setup

Identify G



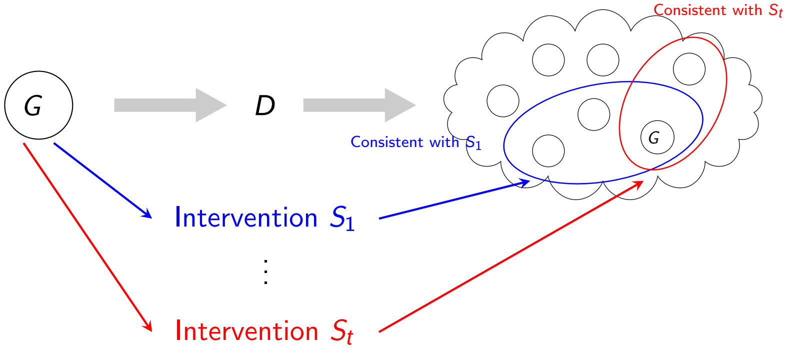
Problem setup (using interventions)

Identify G using as few interventions as possible (minimize t)



Problem setup (using **atomic** interventions)

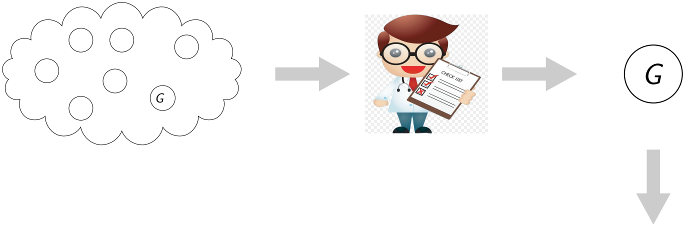
Identify G using as few interventions as possible (minimize t)



Simplifying assumption for this talk:
Each intervention is on a single node, i.e. $|S_1| = \ldots = |S_t| = 1$

Wait a minute... we have domain experts!

Problem solved with zero interventions!

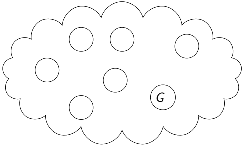


Do stuff with
discovered causal graph G

Wait a minute... we have domain experts!

How do we even check if $G = G$?

~~Problem solved with zero interventions!~~



Experts can be wrong...

Image credit:
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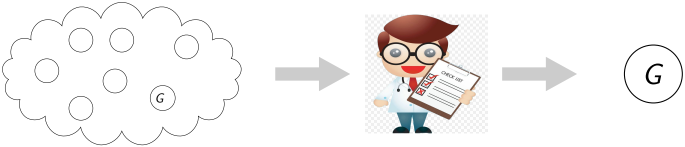
https://img.favpng.com/23/12/11/questionnaire-survey-methodology-png-favpng-CW1Hb5zY6b47rPbAnvWgwEHPK.jpg

The verification problem

Goal: Determine if $G = G$
$\nu(G) =$ minimum number of interventions to answer $G \stackrel{?}{=} G$



We know: Intervening on v orients all arcs incident to v

Trivial solution: Compute minimum vertex cover (MVC) on unoriented arcs! i.e. $\nu(G)$ MVC(unoriented)
(Can be a very bad upper bound!)

Verification: A complete characterization via covered edges

Meek rules $\Rightarrow$ Outperform `MVC`(unoriented)

Surprisingly, enough to compute `MVC` on a *subset of edges*

Covered edges [Chi95]:
$u \to v$ is covered edge $\iff$ $\mathrm{Pa}(u) \setminus \{v\} = \mathrm{Pa}(v) \setminus \{u\}$

Claim: Necessary and sufficient to intervene on `MVC`(covered)
Proof: Simple (but subtle) using the notion of covered edges

Claim: Covered edges form a forest.
Implication: `MVC`(covered) can be computed *exactly* in *linear time*.

Easy re-interpretation of known facts via covered edges

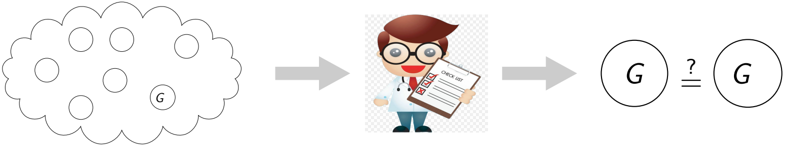
- Covered edges of clique K_n: $v_1 \to v_2, \ldots, v_{n-1} \to v_n$
- Covered edges of a tree: incident edges to root vertex
- Necessity of separating system for non-adaptive interventions
  - [Chi95]: Two graphs are equivalent $\iff$ there is a sequence of covered edge reversals to transform between them.
  - Unoriented edge $\Rightarrow$ Covered edge for *some* DAG in eq. class.
  - Conclusion: any *non-adaptive* search must cut *all* edges.
- Covered edge *cannot* have both endpoints as a sink of any maximal clique $\Rightarrow$ $\nu(G) \leq n - r$ (result of [PSS22]).

(Slide catering to domain experts. If interested, pause to read; Else, skip)

The verification problem ✓

Can determine $G \overset{?}{=} G$

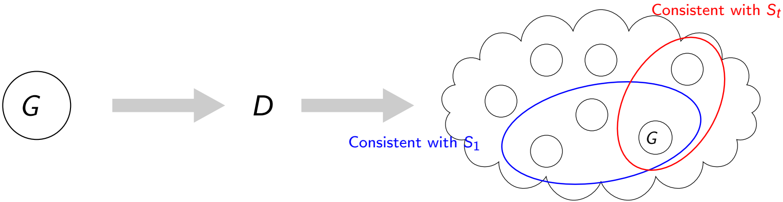
- Using $\nu(G) = \texttt{MVC}(\text{covered})$ interventions
- Computable in polynomial time



What about actually searching for G without the expert?

The adaptive search problem

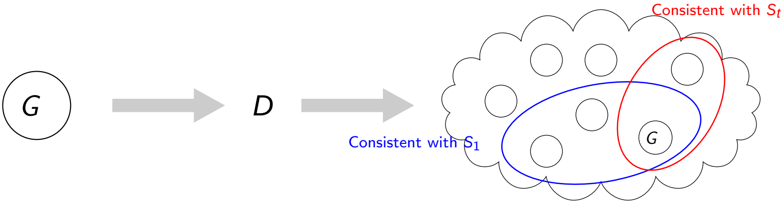
Goal: Identify G using as few interventions as possible



We know that at least $\nu(G)$ interventions is *necessary*

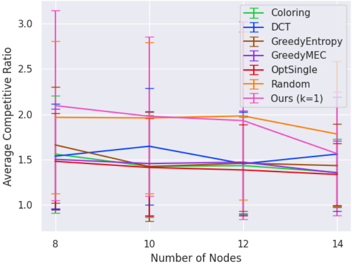
The adaptive search problem

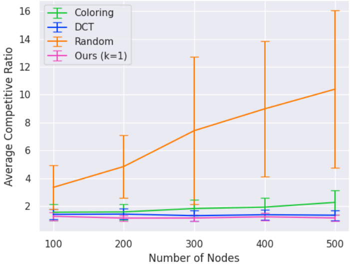
Goal: Identify G using as few interventions as possible



- We know that at least $\nu(G)$ interventions is *necessary*
- Punchline: $O(\log n \quad \nu(G))$ interventions suffice
  - Algorithm: Use chordal graph separators; recurse on subgraphs
  - Analysis: We prove stronger lower bound on $\nu(G)$
- Prior works only have theoretical guarantees on special classes of graphs; The guarantee that we have holds for *any* graph.

Experiments (Atomic search comparision)



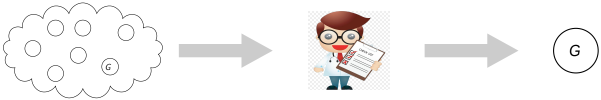


Qualitatively, our algorithm is competitive with the state-of-the-art search algorithms while being 10x faster in some experiments.

Implementation: https://github.com/cxjdavin/verification-and-search-algorithms-for-causal-DAGs

Summary

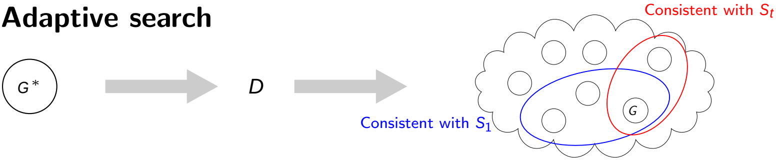
1. **Verification**



Polynomial time *exact characterization* of $\nu(G)$

$\nu(G) = \texttt{MVC}(\text{covered})$ to determine if $G \stackrel{?}{=} G$

2. **Adaptive search**



Polynomial time adaptive search algorithm using interventions

$O(\log n \quad \nu(G \quad))$ suffice for *any general graph*

$\Omega(\log n \quad \nu(G \quad))$ worst case necessary

Natural follow up questions

In this work, we studied *verification* and *search* under an idealized setting with hard interventions and infinite samples.

Soft interventions may be more realistic in certain real-life scenarios (e.g. effects from parental vertices are not completely removed but only altered); see [KJSB19]

Sample complexities also play a crucial role when one has limited experimental budget; see [ABDK18]

We also make standard assumptions such as the Markov assumption, the faithfulness assumption, and causal sufficiency [SGSH00]. Can we remove/weaken these assumptions?

Want to learn more?

Read our paper and/or see our longer talk here:

https://github.com/cxjdavin/verification-and-search-algorithms-for-causal-DAGs/tree/main/talk

More examples to facilitate understanding and explanation of intuition behind some of our techniques, including:

- Why is identifying a set of interventions to fully orient G is equivalent to answering $G \stackrel{?}{=} G$
- A simple concrete example showing why the prior known bounds on $\nu(G)$ is loose.
- Why is $\Omega(\log n \quad \nu(G \quad))$ necessary for search?
- What is our stronger lower bound? How does it work?

Thank you for your kind attention!

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